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RESEARCH ARTICLE

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Linearization and PI Controller Based Ball and Beam System with an Actuator DC motor

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Abstract. This paper presents a revised iteration of the ball and beam system (BBS) that functions as a platform for exploring innovative control methodologies in real-time. The BBS consists of a lengthy beam with a ball situated atop, and the primary objective of the control mechanism is to maintain the ball's position on the beam by manipulating the angular location of the beam. The actuator employed in this study is a direct current (DC) motor. The mathematical model of the system was derived by applying the Newton-Euler equations. Subsequently, the resulting equations were linearized using MATLAB. The efficacy of the proposed PI controller was evaluated using the linearized model of the Ball and Beam system on the MATLAB-SIMULINK platform through careful adjustment of the controller parameters; K_p, K_i, and, K_d. The simulations conducted on SIMULINK and MATLAB demonstrate the proficient real-time performance of the designed proportional and integral (PI) controller.

Keywords: linearization; Simulink ,ball and beam system(BBS);DC motor; Proportional-Integral (PI) controller.

1-Introduction

The BBS is a widely used educational plant in control laboratories worldwide due to its highly nonlinear and unstable nature, rendering it challenging to control. Consequently, it serves as an ideal benchmark for evaluating various advanced control techniques [1,2]. Numerous researchers have extensively investigated and utilized the BBS in their experimentation of novel methodologies. In the study conducted by Authors In[1], the achievement of stabilization and ball location tracking control was successfully accomplished through the implementation of two control schemes: pole placement (PP) and proportional integral derivative (PID) controllers. As stated in[1], the PID controller was developed through the manual tuning approach, whereas, the PP controller was implemented through the MATLAB command. It is noteworthy that both controllers effectively meet the specified design requirements. In their publication in [3], Taifour et al. proposed a PID controller to regulate the position of a ball in a beam system. The controller was developed utilizing two feedback loops, namely the inner and outer loops. The inner loop regulates the angle position of the motor gear, while the outer loop employs the feedback from the inner loop to regulate the ball's position. Furthermore, the research conducted in [4] focuses on the development of a relatively optimal controller (ROC) for the stability of the BBS. This system is a nonlinear, benchmark system that possesses two degrees of freedom. The controller, which exhibits a dynamic construction, is meticulously designed through the resolution of a convex optimization problem.

In [5], the primary objective of the research is to develop a model for the BBS that takes into account nonlinear issues and the effect of coupling . Additionally, the research aims to design controllers that can effectively regulate the position of the ball. To achieve this, a Linear Quadratic Regulator (LQR) is designed, considering both the two Degrees-of-Freedom and the coupling dynamics. in order to accomplish this objective, a Linear Quadratic Regulator (LQR) is formulated, taking into account the dynamics of both the two Degrees-of-Freedom and the coupling. The LQR parameters are meticulously adjusted utilizing the Genetic Algorithm (GA).Additionally, The linearized system is obtained by employing the Jacobian method to approximate the behaviour of the system around its operating point. Furthermore, Meenakshipriya et al. have proposed a design approach for a PID controller of BBS , utilizing the Coefficient Diagram Method (CDM). According to their findings, CDM facilitates the development of a controller that satisfies stability requirements [6].

The paper is organized into four sections. Section 2 provides a comprehensive description of the Ball and Beam System (BBS) and its nonlinear equations of motion, which have been extensively studied in previous research. In Section 3, the implementation of these nonlinear equations in the BBS is explained using Matlab-Simulink. Additionally, the linearized model is derived using the Linmod function, and a design methodology for a Proportional-Integral (PI) controller is introduced. This methodology involves tuning the parameters of a PID controller to optimize the system's performance. The performance evaluation is conducted by analyzing various metrics such as steady state errors, rise time (tr), settling time (ts), and maximum peak overshoot (%Mp). The position of ball tracking control is achieved through manual tuning using Simulink-MATLAB. Section 4 presents the outcomes of the stabilization process for the controlled BBS, which is subsequently followed by a conclusive summary in Section 5.

2.Ball and Beam System

2.1 Ball and Beam Dynamics:

According to [7], a ball is positioned on a beam, enabling it to freely roll along the beam's length. The lever arm is affixed to one end of the beam, while a servo gear is attached to the other. When the servo gear is rotated by an angle θ, the lever modifies the angle of the beam by $\propto$. Consequently, any deviation of the angle from the horizontal position causes the ball to move along the beam, propelled by the force of gravity. In order to enable manipulation of the ball's position within the system, it is imperative to design a dedicated controller. This controller will facilitate the desired adjustments and control over the ball's positioning. The depicted model in Figure 1 represents a ball and beam system.

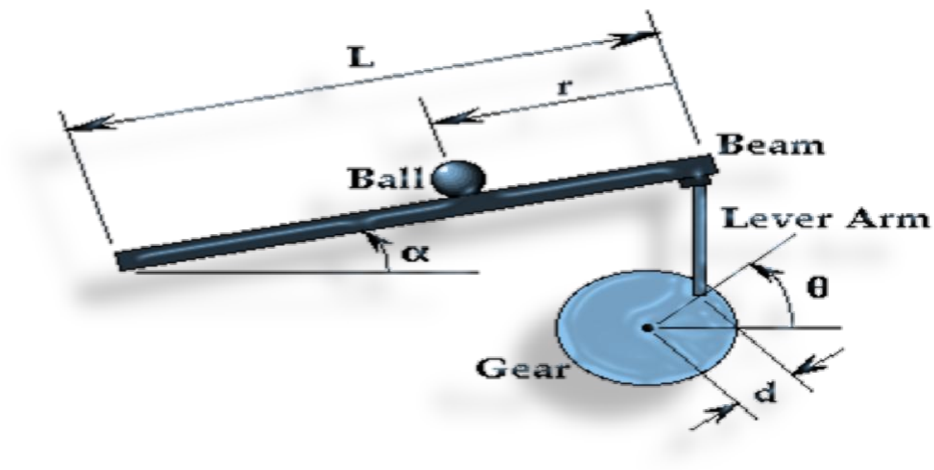

Figure1. Ball and beam system diagram [7,9]

The Lagrangian function, as presented in the work of Taifour et al., is employed to derive the nonlinear equation of motion for the ball [7, 10].

$$\left(\frac{J}{R^2}+m\right)\ddot{r}+mg\sin\alpha-m\,r(\dot{\alpha})^2=0 \qquad (1)$$

The linearization of equation (1) is derived by evaluating the angle of the beam (α = 0), as indicated by equation (2):

$$\left(\frac{J}{R^2}+m\right)\ddot{r}=-mg\propto \qquad (2)$$

The linear equation that provides an approximation of the relationship between the beam angle and the gear angle can be expressed as follows

$$\propto=\frac{d}{L}\ \theta \qquad (3)$$

Substituting α with this value in equation (2) results in:

$$\left(\frac{J}{R^2}+m\right)\ddot{r}=-mg\frac{d}{L}\,\theta \qquad (4)$$

Table 1: Parameters of BBS[8].

Parameter	Description	Value
m	Ball's mass	0.111Kg
g	Gravity acceleration	9.8 (m/s^2)
d	The distance from the centre of the gear to the end point of the moving beam	0.03m
L	The length of the beam	1.0 m
R	Ball's radius	0.015m
J	Inertia moment of the ball	$2*m*R^2/5$

2.2.DC Servo Motor Model

The DC motor enables both translational and rotary motion, as depicted in Figure 2 which showcases the electric circuit of the armature and the rotor diagram.

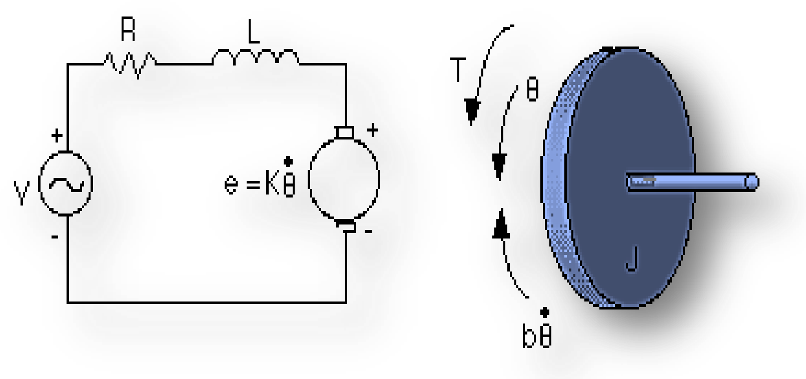

Figure 2: DC motor circuit[8,9]

It is reasonable to assume that the magnetic field remains unchanging, leading to a direct proportionality between the motor torque (T) and the armature current (i) through a constant factor (Kt) known as the torque constant, as represented in Equation (5). This particular type of motor is commonly denoted as an armature-controlled motor in academic literature

$$T = K_t\, i \qquad (5)$$

The relationship between the back electromotive force (emf) and the angular velocity of the shaft (θ) is expressed through a constant factor (Kb), known as the electromotive force constant. This relationship is mathematically represented by Equation (6).

$$e = K_b \ \theta \qquad (6)$$

The motor torque constant (Kt) and the back electromotive force (emf) constant (Kb) are found to be equal, denoted as K. Henceforth, K will be used to represent both the motor torque constant and the back emf constant. Utilizing Figure (2) as a reference, we can derive the following governing equations by applying Newton's second law (Equation 7) and Kirchhoff's voltage law (Equation 8)

$$J\,\ddot{\theta} + b\,\dot{\theta} = Ki \qquad (7)$$

$$L_a \ \frac{di}{dt} + R_a * i = v - K\dot{\theta} \qquad (8)$$

The given equation represents the relationship between various parameters in a system. The moment of inertia of the rotor, denoted as (J), the motor viscous friction constant, denoted as b, the electric inductance, denoted as L_a , the electric resistance, denoted as R_a, and the voltage source, denoted as v, are all involved in this equation.

Table 2 Parameters of DC motor[8].

Parameter	**Value**
Gears ratio(Kg)	
Motor torque constant and the back emf constant (K)	K* Kg=K=0.01
Moment of inertia of the rotor (J)	J=0.01
Constant of the motor viscous friction (b)	b=0.1
Electric inductance of the motor (L_a)	L_a=0.5
Motor electric resistance (Ra)	R_a=1

3.Simulink Model

3.1.Nonlinear Simulink Model of Ball and Motor System:

The BBS structures a single input and four outputs, with the input being the voltage supplied to the DC motor. The outputs consist of the motor current (i), beam tilt angle (∝), gear angle (θ), and ball position (R).

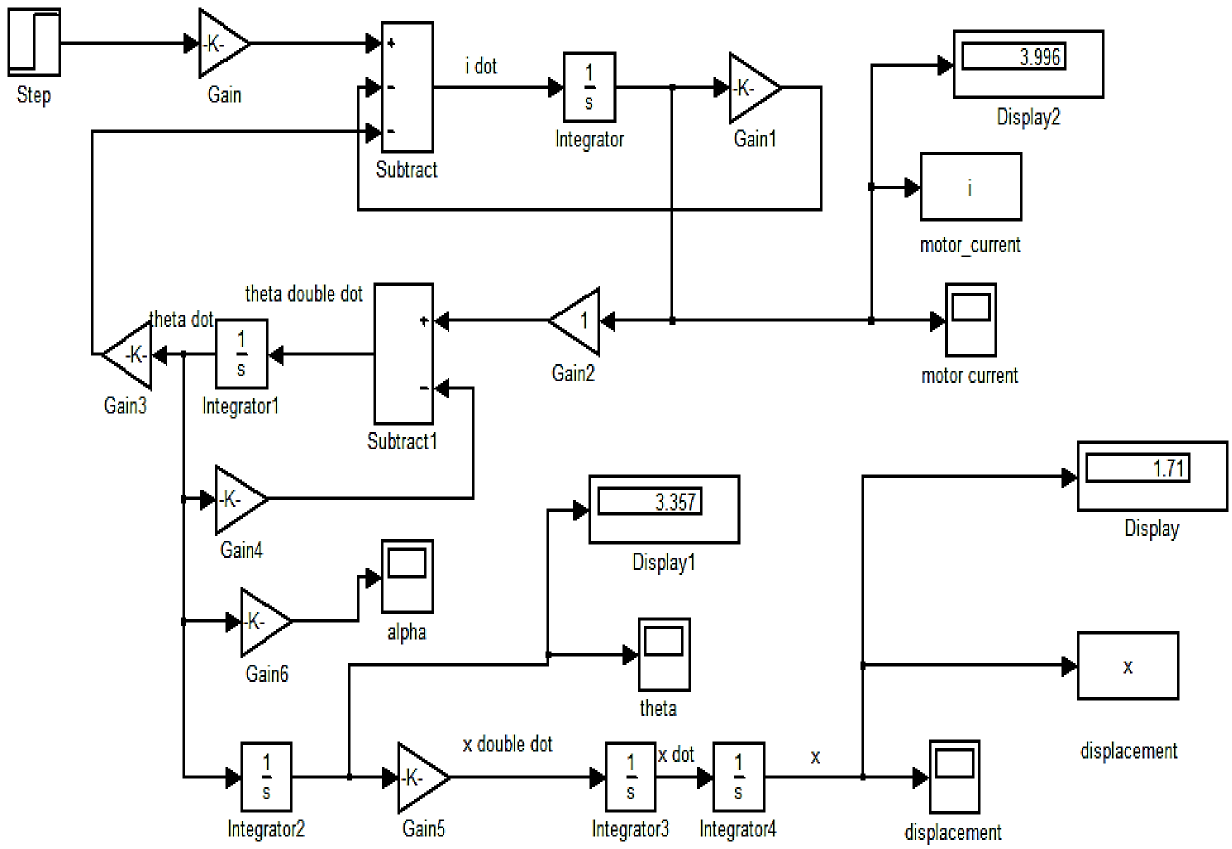

Figure 3. The Simulink Model of the inner structure of the Ball and Beam Physical System

3.2 Linear Simulink Model of Ball and Motor System:

The linearized model is derived from the nonlinear Simulink model using the "linmod function". This function returns the linear state space model, which is depicted in Figure 4 and equation 8 as following :

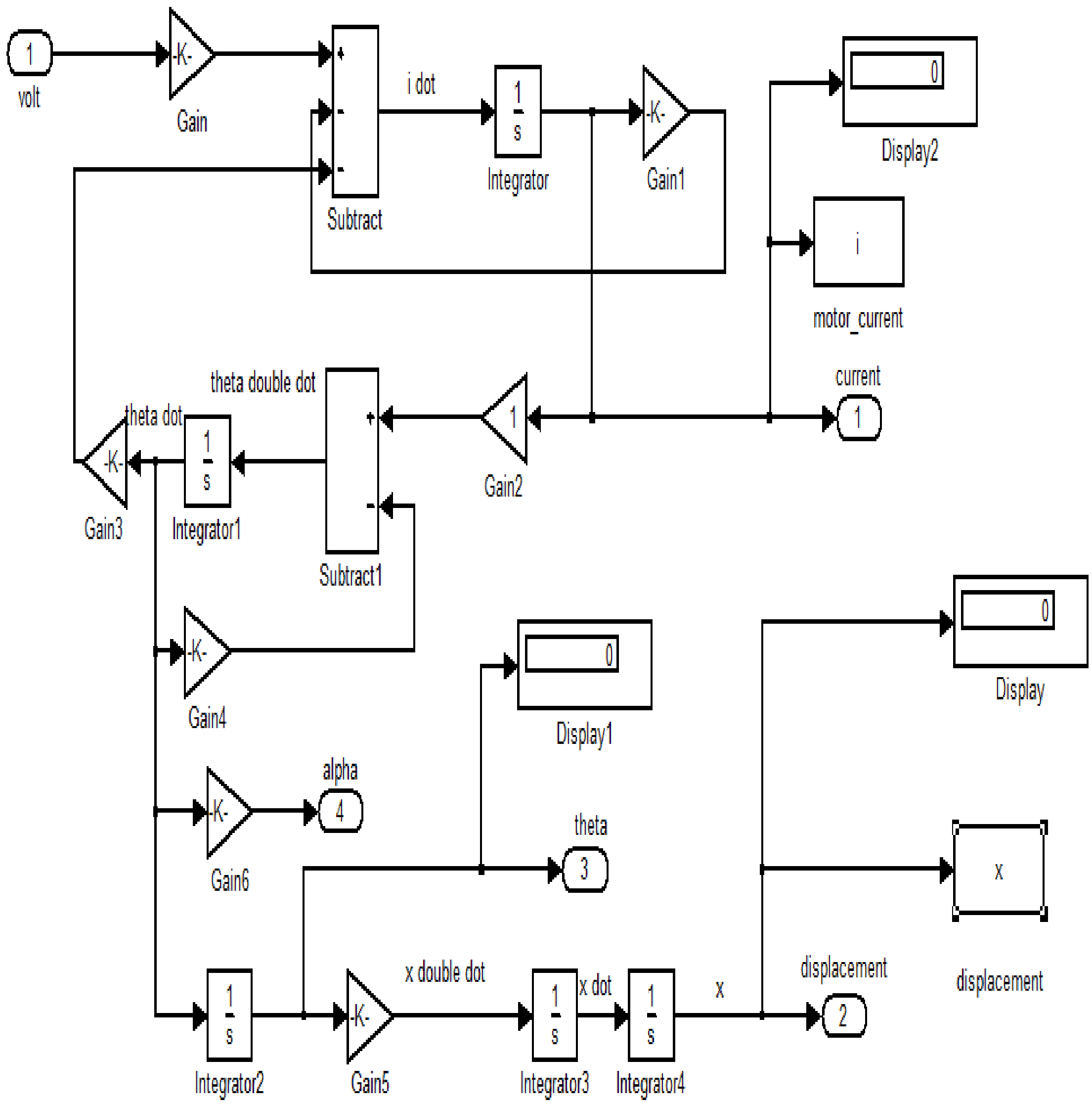

Figure 4. Simulink Model of BBS.

The MATLAB output for the linear state space can be presented as follows:

```
[A,B,C,D]=linmod(''linearised model'')
```

Where A, B, C, and D are defined as follows

$$A = \begin{bmatrix} -2 & 0 & 0 & -0.02 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & -10 & 0 \\ 0 & 0.029 & 0 & 0 & 0 \end{bmatrix}, B \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, C \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0.03 & 0 \end{bmatrix}, and\ \ D \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (8)$$

The Linear state space models without a controller are depicted in Figure 5.

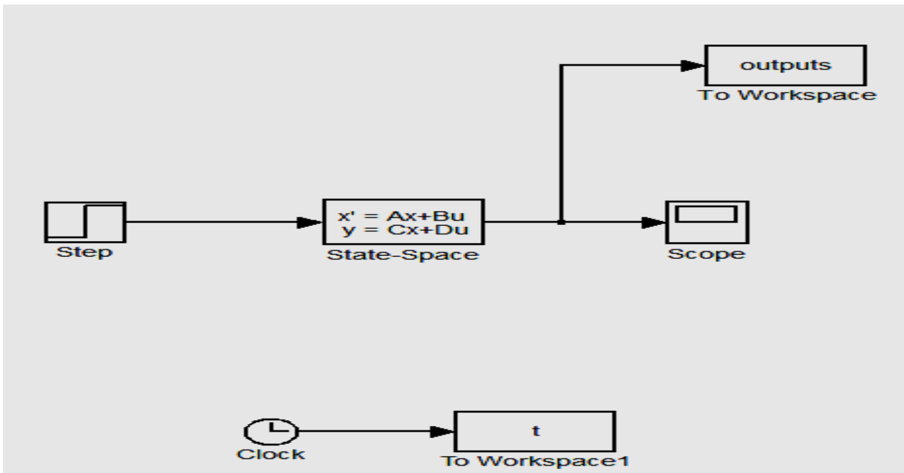

Figure 5. Linear state space models without a controller.

The conversion of Linear state space models without a controller are demonstrated by MATLAB commands to transfer functions as denoted by equation (9) and equation(10)

The conversion of Linear state space models without a controller are demonstrated by MATLAB commands to transfer functions as denoted by equation (8) and equation(9)

```
[A,B,C,D]=linmod("linearised    model");sys=ss(A,B,C,D);Tf(sys)
```
(9)

According to the aforementioned linear state space equations, the transfer functions of BBS can be expressed as follows:

$$G_1(s) = \frac{2\,s + 20}{s^2 + 12\,s + 20.02}$$

$$G_2(s) = \frac{0.08571}{s^5 + 12\,s^4 + 20.02\,s^3}$$

$$G_3(s) = \frac{2}{s^3 + 12\,s^2 + 20.02\,s} \quad (10)$$

$$G_4(s) = \frac{0.06}{s^3 + 12\,s^2 + 20.02\,s}$$

3.3 Linear Simulink Model of Ball and Motor System with PI controller:

The PI controller is utilized for the linearized BBS, as depicted in figure (6). The parameters of the PID controller are adjusted to optimize the system's performance, with Kp=50, Ki=70, and Kd=0, as illustrated in figure (6)

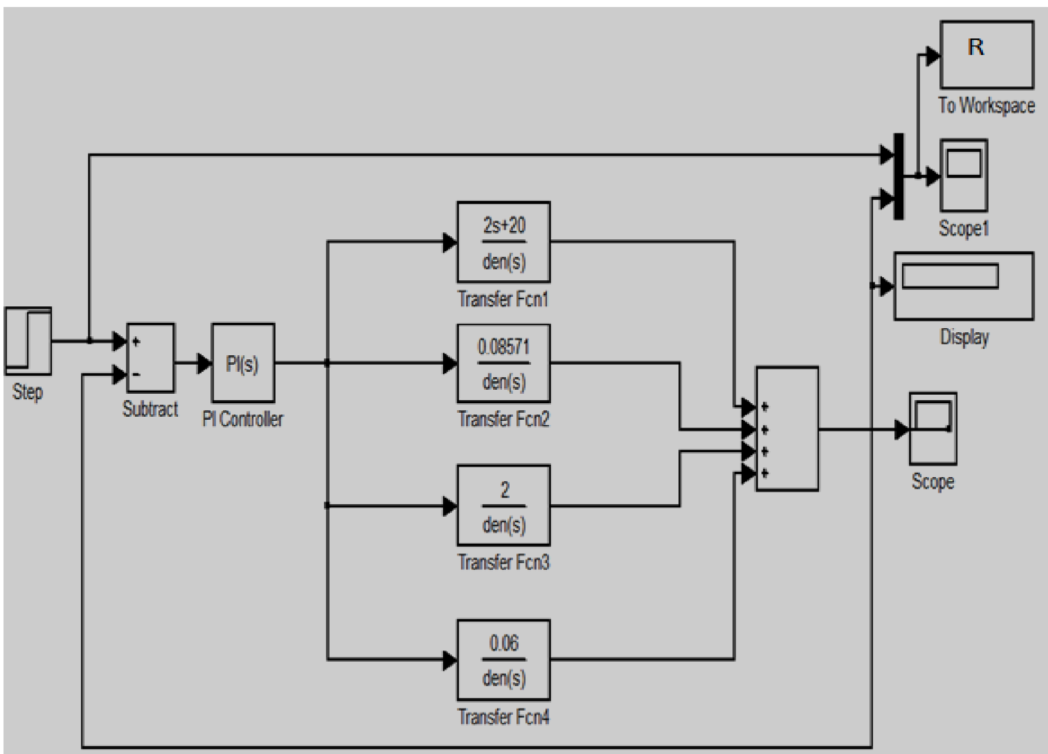

Figure 6 Simulink model of controlled system

4.Results and Discussion

The step response of the nonlinear and uncontrolled BBS is depicted in Figure 7

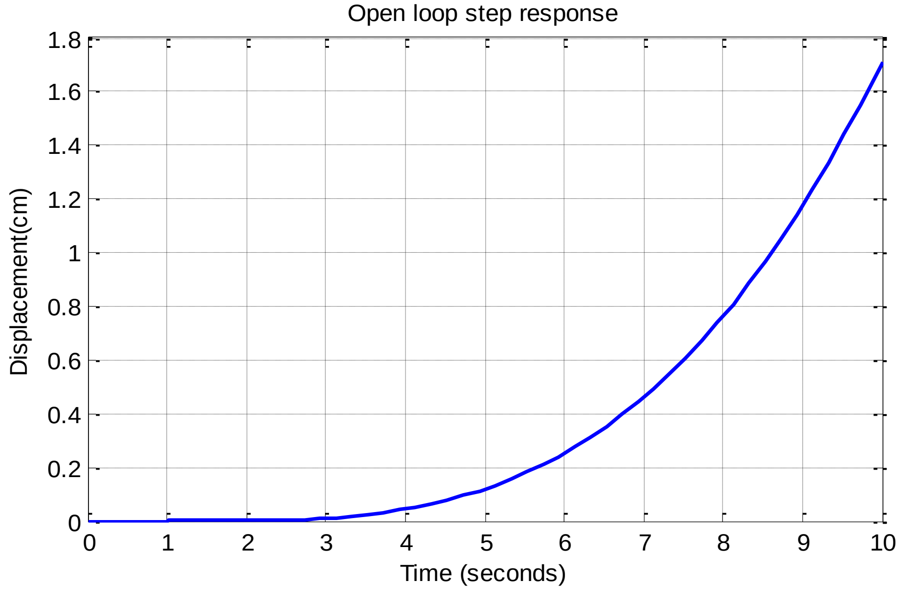

Figure 7 Open loop step response of nonlinear uncontrolled system

The step open-loop response of linearized BBS is depicted in Figure 8.

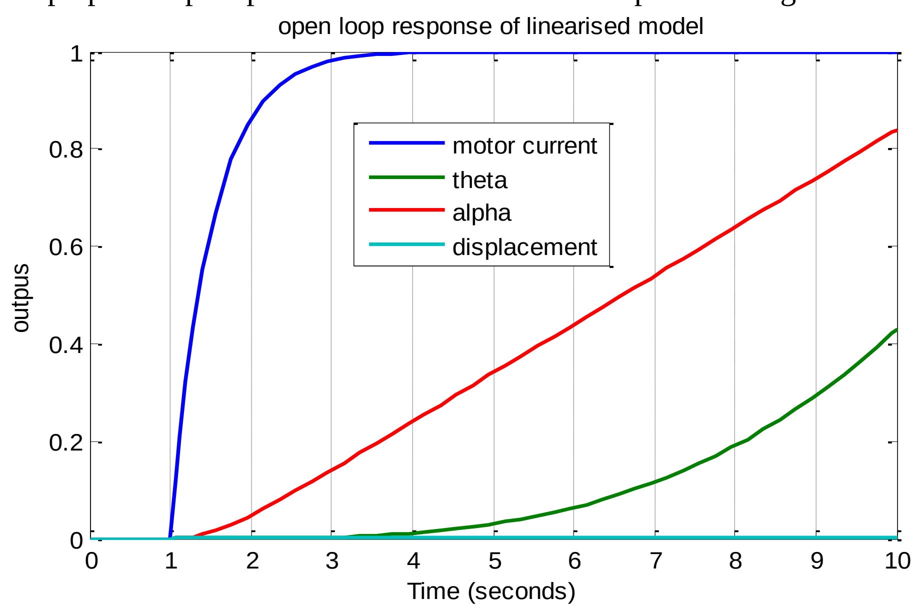

Figure 8 Open loop step response of uncontrolled system.

The performance of the proposed PI controller was evaluated by conducting tests on the linearized model of the Ball and Beam system using the MATLAB-

SIMULINK platform. The controller parameters, including the equivalent time constant (τ) and stability indices, were adjusted to optimize its performance. The results obtained from these tests, as depicted in Figure (9), demonstrate the superior performance of the proposed controller.

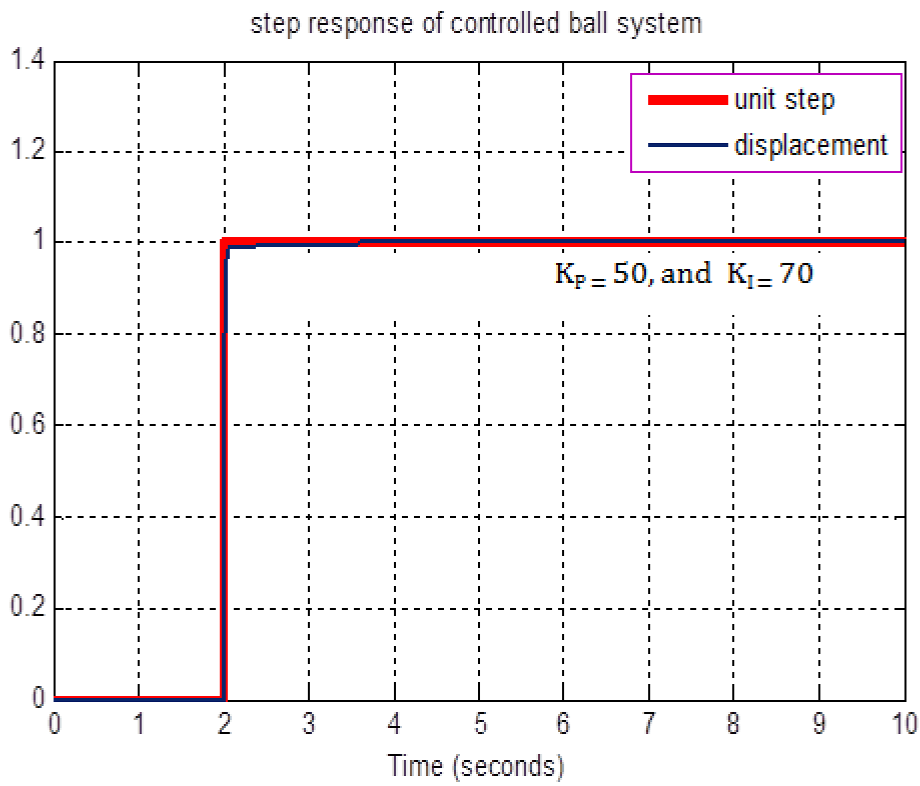

Figure 9 Step response of controlled BBS model .

It can be seen from Figure 9, it is evident that the obtained results demonstrate the superior performance of the response in terms of rise time (t_r) and settling time (t_s), with the absence of any peak overshoot (M_p). Consequently, it can be concluded that the proposed PI control strategy is capable of enhancing the performance of the closed-loop system for any double integrating unstable system.

5.Conclusion

The ball and beam system (BBS) was subjected to a rigorous mathematical analysis, resulting in the creation of a comprehensive model. This model was constructed by utilizing a combination of physical and electrical laws, which were carefully applied to accurately represent the behaviour of the system. The Simulink software is utilized to model nonlinear systems, wherein linearization, parameter design, and evaluation of the PID controller are conducted. The optimal values for the controller parameters (K_p, K_i, and K_d) were derived through the manual tuning method, utilizing the practical model to achieve the most efficient system response. Based on the experimental results, it has been determined that the optimal controller parameters for achieving the most effective response from the system are as follows: Kp=50, Ki=70, and Kd=0. The system's precision was evaluated through the manipulation of the ball's placement. A PI controller was utilized to design the controller for the double integrating unstable system.

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